X. Epilogue

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I.9 (Action vs state-labelled models)

c) Bisimulation and Branching Bisimulation on KS

I.10 Semantic vs. logical equivalence

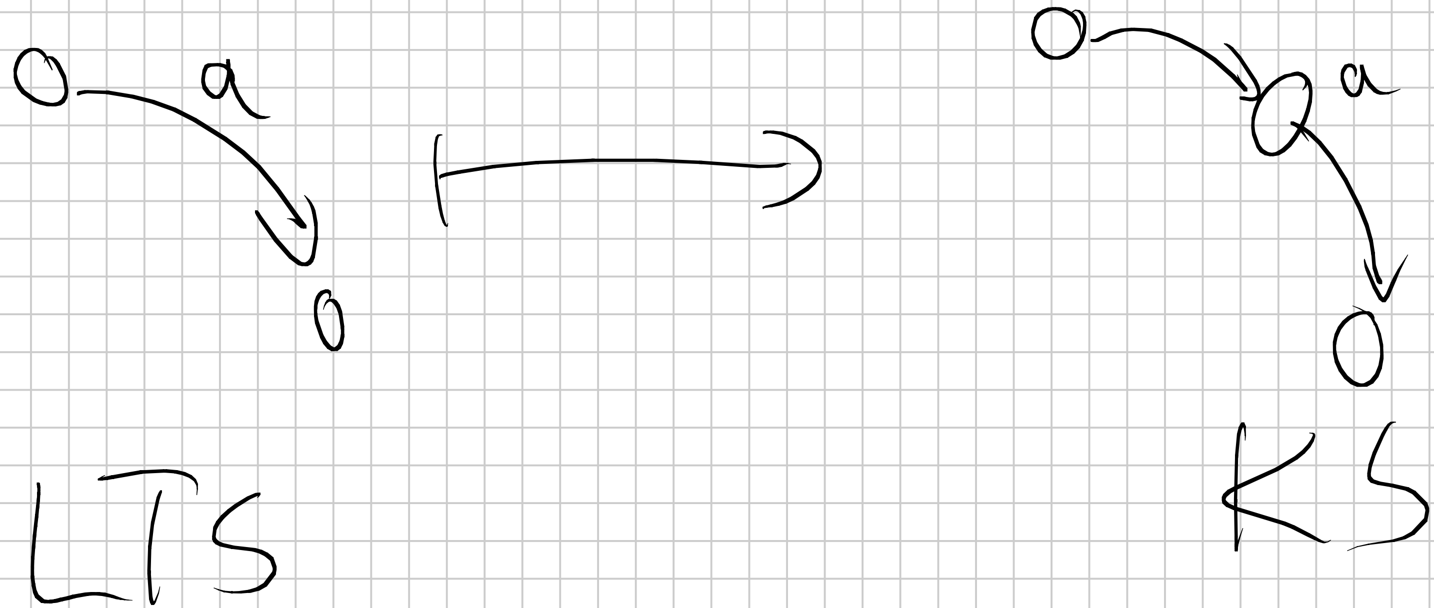
IV. n.1: Borel space over paths in DTMC

IX.3. Axiom-example

I.9.A) KS $\overset{?}{\Longleftrightarrow}$ LTS

Intuitive idea:

Construct morphisms between both models.



[De Nicola, Vaandrager] [ACM 86]

I.9.C Bisimulation and Branching Bisimulation on KS

Def: A relation B on a KS $(S, I, \longrightarrow, L)$ is a bisimulation, if $(s,t) \in B$ implies

i) $L(s) = L(t)$

ii) $s \longrightarrow s'$ implies $\exists t' : t \longrightarrow t' \wedge (s',t') \in B$

iii) $t \longrightarrow t'$ implies $\exists s' : s \longrightarrow s' \wedge (s',t') \in B$

(Def $\sim$ as usual)

I.10 Semantic vs. logical equivalence

Def For $\mathcal{M} = (S, I, \to, L)$ a Kripke structure, we say that $s, t \in S$ are CTL-equivalent, iff $\forall \Phi \in CTL$

$$\mathcal{M}, s \models \Phi \iff \mathcal{M}, t \models \Phi$$

LTL-equivalence etc are defined analogously.

Thm: $s \sim t$ iff s and t are CTL-equivalent.

Proof: "$\supseteq$"

We show that $\mathcal{B}$ is a bisimulation, where

$$\mathcal{B} = \{(s,t) \mid s,t \text{ satisfy the same CTL-formula}\}$$

We observe that $\mathcal{B}$ is an equivalence relation.

We only show (i) and (ii) of Bisimulation def.

(i) Let $(s,t) \in \mathcal{B}$. $s \models \bigwedge_{a \in L(s)} a \wedge \bigwedge_{a \notin L(s)} \neg a$, since s,t are CTL-equivalent, we also have $t \models \bigwedge_{a \in L(s)} a \wedge \bigwedge_{a \notin L(s)} \neg a$

Thus: $a \in L(s) \iff a \in L(t)$

(ii) We consider the equivalence classes $C \in S/_{D}$, and construct characteristic formula Φ_C such that $Sat(\Phi_C) = C$.

How do we do this?

For each pair C, D of equivalence classes we choose a CTL-formula $\Phi_{C,D}$ such that

$$Sat(\Phi_{C,D}) \supseteq C, \text{ but } Sat(\Phi_{C,D}) \cap D = \emptyset$$

These formula must exist by construction of $\mathcal{B}$

Then we set $\Phi_C = \bigwedge_{\substack{D \in S/R \\ D \neq C}} \Phi_{C,D}$

Now assume

$$s \longrightarrow s' \quad \text{and} \quad s' \in C \quad \text{for some } C \in S/_B$$

Thus $\quad s \models EX\, \Phi_C$

By assumption

$$t \models EX\, \Phi_C$$

and hence

t' must exist, such that

$$t \longrightarrow t' \quad \text{and} \quad t' \in C$$

whence $\quad (s', t') \in B \quad$ follows.

$\square$

"$\subseteq$" Assume $s \sim t$, take Φ, an arbitrary CTL-formula. Proceed by induction on the structure of Φ.

Base case:

$\Phi = \text{true}$ ✓

$\Phi = a$, i.e. $s \models a \Leftrightarrow a \in L(s) \Leftrightarrow a \in L(t) \Leftrightarrow t \models a$

$s \not\models a \Leftrightarrow a \notin L(s) \Leftrightarrow a \notin L(t) \Leftrightarrow t \not\models a$

Induction

$\Phi = \Phi_1 \wedge \Phi_2$ ✓

$\Phi = \neg \psi$ ✓

$\Phi = A X \psi$

We prove that

$$s \models AX\psi \iff t \models AX\psi$$

Assume $s \models AX\psi$, but $t \not\models AX\psi$

Thus $\exists t'$ such that $t \longrightarrow t'$ and $t' \not\models \psi$

Since (s,t) are bisimilar there must be s' such that

$s \longrightarrow s'$, and (s',t') are bisimilar. By induction

$s' \not\models \psi$, but this contradicts

$s \models AX\psi$

Todo:

$\Phi = EX\psi$

$\Phi = A(\Phi_1 \cup \Phi_2)$, $E(\Phi_1 \cup \Phi_2)$

$\square$