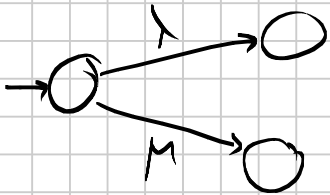
V. Simulation.

V.1. Semi Markov Processes

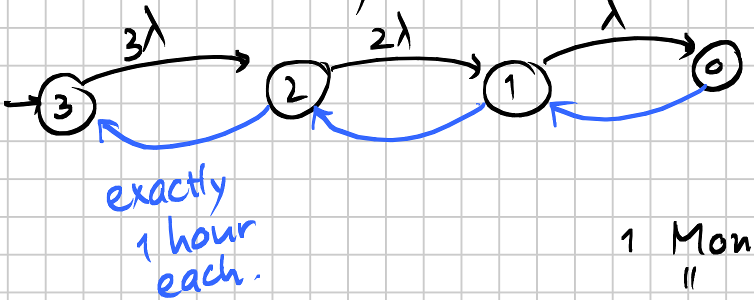
CTMC: States; initial states;
transitions labelled with rates;
state labelling.

bestimmen den Zeitpunkt
des Übergangs. always exponential.



etc.

Triple redundant system:



1 Zeiteinheit = 1 Stunde.

EXP[λ]:

1 Monat = Erwartungswert $= \frac{1}{\lambda}$

$\parallel$ ca. $30 \cdot 24$ h

$\Rightarrow \lambda = \frac{1}{30 \cdot 24}$

Def. Semi-Markov Process: (S, P, Q, L, s_0)

S – states with initial state $s_0 \in S$.

$P: S \times S \longrightarrow [0,1]$ — transition probabilities

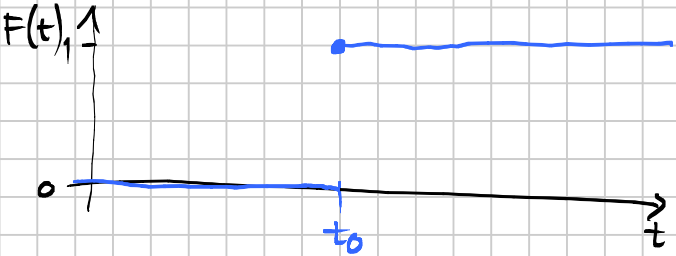
$Q: S \times S \longrightarrow (\mathbb{R}_0^+ \longrightarrow [0,1])$ – for each transition, a cumulative distribution fction

$L: S \longrightarrow 2^{AP}$ – state labelling

In a CTMC, all cdf were exponential:

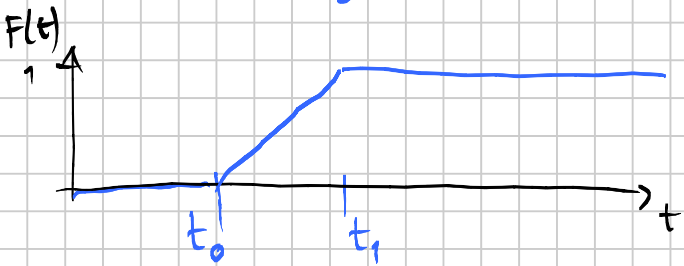
$$F(t) = P(X \leq t) \quad , \; X = \text{Verweildauer.}$$

In a SMP, other cdfs are allowed, e.g.:



deterministische cdf:

$$F(t) = \begin{cases} 0 & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$



uniforme cdf.

Semantics of a SMP.

1. Upon entering a state s, the next state s' is selected with probability $P(s, s')$.
2. The sojourn time in state s is determined by $Q(s, s')$. (kitchen timer)

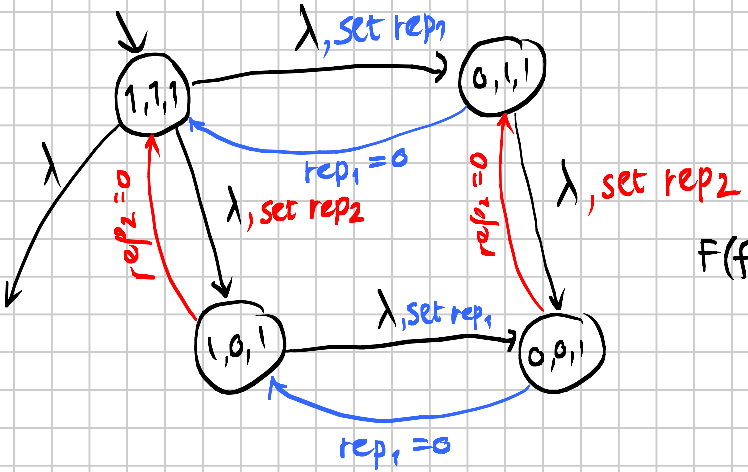
Q: Is this a good model for the triple redundant example?

A: No, the system forgets that the technician has already done some work to replace the first computer when the second one fails.
(No problem in CTMC $\Leftarrow$ memorylessness)

V. 2. Generalised Semi-Markov Processes.

Events:

1. Each event has its own cdf.
2. If an event becomes active (i.e. it is bound to happen), pick a time according to its cdf, and set its kitchen timer.
3. If something happens (and the event remains active), do not change its kitchen timer.
4. If a timer goes off, take the corresponding transition.



$\mathrm{next}(1,1,1,\mathrm{fail}_1) = (0,1,1)$
usw.

$F(\mathrm{fail}_2) = F(\mathrm{fail}_1) = \mathrm{EXP}(\lambda)$

$t \mapsto 1 - e^{-\lambda t}$

$F(\mathrm{rep}_1) = \mathrm{DET}(1\,h)$
$\parallel$
$F(\mathrm{rep}_2)$

$S = \{(1,1,1), (0,1,1), (1,0,1) \ldots \}$ $\quad s_0 = (1,1,1)$

$E = \{\mathrm{rep}_1, \mathrm{rep}_2, \mathrm{rep}_3, \mathrm{fail}_1, \mathrm{fail}_2, \mathrm{fail}_3\}$

$\mathrm{active}(1,1,1) = \{\mathrm{fail}_1, \mathrm{fail}_2, \mathrm{fail}_3\}$
$\mathrm{active}(1,0,1) = \{\mathrm{fail}_1, \mathrm{fail}_3, \mathrm{rep}_2\}$ usw.

GSMP: $(S, E, active, next, F, s_0)$

S	states
E	events
$active: S \to 2^E$	assigns a set of active events to each state.
$next: S \times E \to S$	assigns a next state (if an event happens)
$F: E \to (\mathbb{R}_0^+ \to [0,1])$	assigns a cdf to each event
s_0	initial state

Nondeterminism in GSMPs?

└ there is a choice, but we cannot assign probabilities.

At time 1, we have a nondeterministic choice between going left and going right.

Nondet. cannot be simulated!

Solution: forbid non-continuous cdfs.

(a little too restrictive: the example of the triple redundant system does not contain nondet.)

is nondeterministic with probability 0.